Outage Information Rate of Spatially Correlated Multi-Cluster Scattering MIMO Channels

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Outage Information Rate of Spatially Correlated Multi-Cluster Scattering MIMO Channels†

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***Abstract*—A one-sided spatially-correlated multi-cluster scattering Rayleigh MIMO channel is considered in this work and its outage probability is derived in an analytic form based on Meijer function determinants. First, the spatially-uncorrelated case is addressed and the Moment Generating Function (MGF) of the information rate is expressed in an analytic closed-form. The MGF is then used to obtain the outage probability. A few special cases are addressed to provide a confirmation of the analytic results. Next, the MGF in the one-sided spatially correlated case is derived with the constraint of distinct positive spatial correlation eigenvalues. Numerical results are included to provide confirming evidence of the analytic results. These results are then used to assess the outage probability degradation due to spatial correlation in a selected example.**

I. Introduction

According to recent statistical analyses [1], the overall mobile data traffic is expected to grow more than linearly in the next few years and exceed an overall amount of 30 EB per month by 2020. The traffic demand growth is largely due to the ever increasing use of the wireless channel to convey video information to stream popular contents such as news reports or entertainment programs. This is spurring an increasing demand for traffic coverage and capacity, especially from smartphone devices and machine-to-machine communications. Eventually, it may lead current networks to become unable to meet the traffic demand and has motivated the development of new technologies in the framework of a yet unknown fifth generation (5G) wireless communication system, which is expected to be deployed around 2020 [2].

The interest in 5G technologies has pushed a great deal of attention on Pico-Cellular networks, which have been proposed to improve the traffic coverage in small areas like train stations, office buildings, shopping malls, airports, trains, and aircrafts. For these networks, multi-layered scattering channel models are considered to describe the indoor signal propagation between different floors in a building or through walls separating a common environment [3, ch. 13]. The ergodic capacity of multiple-input multiple-output (MIMO) channels based on multi-layered scattering has been derived in [4], [5] in the case of equal and unequal number of scatterers in each cluster, respectively. Moreover, the ergodic capacity scaling law has been established in [7]. More recently, the outage information rate has been derived under the assumption of Orthogonal Space–Time Block Code (OSTBC) transmission in [8].

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In the current work we extend the analysis to encompass the outage information rate of a multi-layered scattering MIMO channel *without any constraints* on the coding scheme. Our approach is based on the results from [5], [6], [9] concerning the joint positive singular value distribution of the product of iid Rayleigh MIMO channel matrices modelling a multi-layered scattering MIMO channel. The authors of [5] focused their work on the derivation of the ergodic capacity and derived expressions for the marginal distribution of the positive singular values of the Wishart matrix derived from the multi-layered scattering channel matrix. However, their results can also be used to derive an analytic expression of the Moment-Generating Function (MGF) of the capacity. In turn, the MGF leads to the derivation of the outage probability, thereby completing the picture of the capacity analysis.

As already mentioned, an outage analysis was carried out in [8] but it was limited to the case of OSTBC transmission and did not apply in general. The authors exploited a peculiar property of OSTBCs consisting in the fact that their capacity depends on the squared Euclidean norm of the channel matrix so that they obtained its MGF and used it for their outage analysis. Here, we resort to a different approach based on the MGF of the MIMO channel capacity without specific assumptions on the code used.

The paper is organized as follows. Section II introduces the multi-layered scattering Rayleigh MIMO channel model based on the product of size-compatible iid Rayleigh channel matrices. Then, it reports the relevant results from [4], [9] concerning the joint distribution of the positive singular values of the channel matrix itself (by which they derived the marginal distribution and an expression for the ergodic capacity). The MGF of the capacity is therefore derived by integration over the unitary group. Two special cases, corresponding to zero and one intermediate scattering clusters, are considered in detail, the former of them being used to check the compatibility with existing literature results. Finally, the outage probability is derived according to methods adapted from [10]. Section III addresses the case of one-sided spatial correlation at the receive or transmit side. Obviously the transmit-side correlation case encompasses that of correlated transmitted signal. This work extends the results from [11] where only the ergodic capacity was addressed. Numerical results are presented in Section IV addressing the spatially uncorrelated and correlated cases for specific MIMO systems. The results are presented along with Monte-Carlo simulation points for numerical validation. Finally, concluding remarks are provided in Section V.

II. Channel model

We consider a MIMO channel model described by the equation

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{z} \tag{1}$$

where the channel matrix is written as

$$\boldsymbol{H} = \boldsymbol{H}_M \boldsymbol{H}_{M-1} \cdots \boldsymbol{H}_1 \tag{2}$$

and the matrices $\boldsymbol{H}_m \in \mathbb{C}^{N_m \times N_{m-1}}$ contain iid random entries distributed as $\mathcal{CN}(0,1)$ (Ginibre matrices). Therefore, $\boldsymbol{H} \in \mathbb{C}^{N_M \times N_0}$, so that $n_R = N_M$ and $n_T = N_0$ are the number of receive and transmit antennas, respectively. Moreover, we assume that the entries of $\boldsymbol{x}$ and $\boldsymbol{z}$ are iid and distributed as $\mathcal{CN}(0,\gamma)$ and $\mathcal{CN}(0,1)$, respectively.

A. Positive singular value distribution

The following results have been shown in [5], [9]:

- The positive singular values of (2) are identical to the singular values of the matrix product
$$\widetilde{\boldsymbol{H}} = \widetilde{\boldsymbol{H}}_M \widetilde{\boldsymbol{H}}_{M-1} \cdots \widetilde{\boldsymbol{H}}_1$$
where the matrices $\widetilde{\boldsymbol{H}}_m$ are square matrices of size
$$N_{\min} \triangleq \min_m N_m \tag{3}$$
obtained by the factorization
$$\boldsymbol{H}_m = \boldsymbol{U}_m \begin{pmatrix} \widetilde{\boldsymbol{H}}_m & \boldsymbol{A}_m \\ \boldsymbol{0} & \boldsymbol{B}_m \end{pmatrix} \boldsymbol{U}_{m-1}^{-1}$$
where, for $m = 1, \ldots, M$,
$\boldsymbol{U}_m \in \mathbb{C}^{N_m \times N_m}$ is a unitary matrix: $\boldsymbol{U}_m^{-1} = \boldsymbol{U}_m^{\mathsf{H}}$
$\widetilde{\boldsymbol{H}}_m \in \mathbb{C}^{N_{\min} \times N_{\min}}$
$\boldsymbol{A}_m \in \mathbb{C}^{N_{\min} \times (N_{m-1} - N_{\min})}$
$\boldsymbol{B}_m \in \mathbb{C}^{(N_m - N_{\min}) \times (N_{m-1} - N_{\min})}$
- Based on the previous factorization, the joint distribution of the positive singular values of $\boldsymbol{H}$ is independent of the ordering of the matrix sizes N_m, so that they can be arbitrarily permuted without changing the resulting joint distribution. This fact is implicitly proved in [9] and explicitly stated in [8, Sec. III-A].
To fix ideas, it will be assumed that the right-most matrix size, N_0, is equal to $N_{\min}$. This can be obtained, for example, by sorting the sequence $(N_M, N_{M-1}, \ldots, N_1, N_0)$ in nonincreasing order.
- Assuming $N_0 = N_{\min}$, the joint pdf of the eigenvalues $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_{N_0})$ of $\boldsymbol{H}\boldsymbol{H}^{\mathsf{H}}$ is given by [6]: $p(\boldsymbol{\lambda}) =$
$$\frac{\Delta(-\boldsymbol{\lambda})}{C_M} \det\left[G_{0,M}^{M,0}\left(\left. \begin{matrix} - \\ \nu_M, \ldots, \nu_2, \nu_1 + i - 1 \end{matrix} \right| \lambda_j \right) \right]_{i,j=1}^{N_0} \tag{4}$$
where
$$\Delta(\boldsymbol{\lambda}) = \det(\lambda_i^{N_0 - j})_{i,j=1}^{N_0} = \prod_{i<j} (\lambda_i - \lambda_j),$$
$$C_M = N_{\min}! \prod_{n=1}^{N_0} \prod_{m=0}^{M} \Gamma(n + \nu_m),$$
and the function $G_{0,M}^{M,0}(\cdots)$ is a Meijer-G function defined in [13, 9.30].

B. Moment-Generating Function

By using the joint pdf in (4) we obtain the MGF of the mutual information of the MIMO channel

$$\begin{aligned} \mathcal{I} &= \log\det(\boldsymbol{I} + \gamma \boldsymbol{H}\boldsymbol{H}^{\mathsf{H}}) \\ &= \sum_{i=1}^{N_{\min}} \log(1 + \gamma\lambda_i) \end{aligned} \tag{5}$$

as follows:

$$\begin{aligned} \Phi_{\mathcal{I}}(s) &\triangleq \mathbb{E}[\exp(-s\mathcal{I})] \\ &= \mathbb{E}\left[\prod_{i=1}^{N_{\min}} (1 + \gamma\lambda_i)^{-s} \right] \end{aligned} \tag{6}$$

Notice that the SNR can be obtained as follows:

$$SNR = \frac{\gamma \mathbb{E}[\|\boldsymbol{H}\|^2]}{n_R} = \frac{\gamma \prod_{m=0}^{M-1} N_m}{N_M}.$$

Applying the Andreief identity [12]

$$\begin{aligned} &\int \det(A_i(\lambda_j))_{i,j=1}^m \det(B_i(\lambda_j))_{i,j=1}^m d\boldsymbol{\lambda} \\ &= m! \det\left(\int A_i(\lambda) B_j(\lambda) d\lambda \right)_{i,j=1}^m \end{aligned} \tag{7}$$

we obtain the MGF as

$$\begin{aligned} \Phi_{\mathcal{I}}(s) = \frac{1}{\prod_{n=1}^{N_0} \prod_{m=0}^{M} \Gamma(n + \nu_m)} \det\Bigg[\int_0^\infty \lambda^{i-1} \\ \times (1 + \gamma\lambda)^{-s} G_{0,M}^{M,0}\left(\left. \begin{matrix} - \\ \nu_M, \ldots, \nu_2, \nu_1 + j - 1 \end{matrix} \right| \lambda \right) d\lambda \Bigg]_{i,j=1}^{N_0} \end{aligned} \tag{8}$$

where $\nu_m \triangleq N_m - N_{\min}$. Then, by applying [13, 7.811.5] and [13, 9.31.5] we get the following closed-form expression of the integrals in (8):

$$\begin{aligned} &\int_0^\infty \lambda^{i-1} (1 + \gamma\lambda)^{-s} G_{0,M}^{M,0}\left(\left. \begin{matrix} - \\ \nu_M, \ldots, \nu_2, \nu_1 + j - 1 \end{matrix} \right| \lambda \right) d\lambda \\ &= \frac{\gamma^{-i}}{\Gamma(s)} G_{1,M+1}^{M+1,1}\left(\left. \begin{matrix} 1 - i \\ s - i, \nu_M, \ldots, \nu_2, \nu_1 + j - 1 \end{matrix} \right| \frac{1}{\gamma} \right) \\ &= \frac{1}{\Gamma(s)} G_{1,M+1}^{M+1,1}\left(\left. \begin{matrix} 1 \\ s, \nu_M + i, \ldots, \nu_2 + i, \nu_1 + i + j - 1 \end{matrix} \right| \frac{1}{\gamma} \right) \end{aligned} \tag{9}$$

C. Special case: $M = 1$, direct scattering

Here we review the special case of $M = 1$, which corresponds to the standard iid Rayleigh MIMO channel. This case has already been studied in the literature and the MGF has been derived in [14] and reported in [15]. We apply the results from our general multi-cluster scattering framework to derive the direct scattering MGF in order to check the correctness of our approach.

Let us assume that $N_1 \geq N_0$ so that $N_0 = N_{\min}$. If $N_1 < N_0$ we simply exchange the two matrix sizes.

Accordingly, we set $\nu_1 = N_1 - N_0$ and obtain from (9):

$$\begin{aligned}\int_0^\infty & \lambda^{i-1}(1+\gamma\lambda)^{-s} G_{0,1}^{1,0}\left(\begin{matrix} - \\ N_1 - N_0 + j - 1\end{matrix}\middle|\lambda\right) d\lambda \\ &= \int_0^\infty \lambda^{N_1-N_0+i+j-2}(1+\gamma\lambda)^{-s}\mathrm{e}^{-\lambda} d\lambda \\ &= \frac{1}{\Gamma(s)} G_{1,2}^{2,1}\left(\begin{matrix} 1 \\ s, N_1 - N_0 + i + j - 1\end{matrix}\middle|\frac{1}{\gamma}\right) \quad (10)\end{aligned}$$

This result agrees with [14, eq. (25)] and [15, Th. 2.14] and confirms the validity of our approach.

D. Special case: $M = 2$, single-cluster scattering

Let us consider a single-cluster scattering MIMO system characterized by n_T transmitting antennas, n_S scattering elements in the intermediate cluster, and n_R receiving antennas. Let us sort the vector (n_T, n_S, n_R) as (N_2, N_1, N_0) with the property that $N_0 = \min\{N_2, N_1, N_0\}$. Then, setting $\nu_2 = N_2 - N_0$ and $\nu_1 = N_1 - N_0$, we obtain from (9):

$$\begin{aligned}\int_0^\infty & \lambda^{i-1}(1+\gamma\lambda)^{-s} G_{0,2}^{2,0}\left(\begin{matrix} - \\ \nu_2, \nu_1 + j - 1\end{matrix}\middle|\lambda\right) d\lambda \\ &= \int_0^\infty \lambda^{(\nu_2+\nu_1+2i+j-3)/2}(1+\gamma\lambda)^{-s} K_{1-j-\nu_1+\nu_2}(2\sqrt{\lambda}) d\lambda \\ &= \frac{1}{\Gamma(s)} G_{1,3}^{3,1}\left(\begin{matrix} 1 \\ s, \nu_2 + i, \nu_1 + i + j - 1\end{matrix}\middle|\frac{1}{\gamma}\right) \quad (11)\end{aligned}$$

where $K_\nu(z)$ is the modified Bessel function of the second kind [13, 8.432.1]. Previously, the outage probability has only been characterized in the asymptotic regime of $N_0, N_1, N_2 \to \infty$, with fixed ratios N_2/N_1 and N_1/N_0 in [16].

E. Outage probability

From the related MGF $\Phi_\Delta(s) \triangleq \mathrm{e}^{sR}\Phi_{\mathcal{I}}(s)$ one can easily obtain the outage probability (see [10]):

$$\begin{aligned}P_{\mathsf{out}}(R) &\triangleq P(\mathcal{I} < R) \\ &= \frac{1}{2\pi\,\mathrm{j}} \int_{c-\mathrm{j}\infty}^{c+\mathrm{j}\infty} \Phi_\Delta(s)\frac{ds}{s} \\ &\approx \frac{1}{2L}\sum_{k=1}^{L} \mathsf{Re}\Big\{(1-\mathrm{j}\,\tau_k)\Phi_\Delta(c(1+\mathrm{j}\,\tau_k))\Big\}\end{aligned}$$

where c must be in the convergence region of $\Phi_\Delta(s)$ and, in the last approximation, $\tau_k \triangleq \tan((2k-1)\pi/(4L))$. The approximation improves as L increases. In general, $L = 32$ is sufficiently large for practical purposes.

In order to improve the numerical stability of this approach it is convenient to choose c in order to minimize $\Phi_\Delta(c)$. In fact, this is a consequence of the well-known Chernoff bound [17]:

$$P(\Delta < 0) \leq \min_{c>0} \Phi_\Delta(c).$$

The minimization of $\Phi_\Delta(c)$ reduces the amplitude of the oscillations of the function $\Phi_\Delta(c + \mathrm{j}\,t)$ and hence makes the calculation of the integral more stable from a numerical point of view.

III. Impact of Spatial Correlation

In this section we address the presence of spatial correlation in the multi-cluster scattering MIMO channel. The problem was addressed in [11], where the author derived the ergodic capacity when the spatial correlation affects the receiver only and the number of receive antennas coincides with the minimum matrix size $N_{\min}$. The results of [11] do not apply directly to the cases when $n_R > N_{\min}$ or $n_T > N_{\min}$.

Here, we consider the following spatially-correlated multi-cluster scattering MIMO channel matrices:

$$\begin{aligned}\boldsymbol{H}_{rx} &= \boldsymbol{\Sigma}_{rx}^{1/2}\boldsymbol{H}_M \cdots \boldsymbol{H}_1 &(12)\\ \boldsymbol{H}_{tx} &= \boldsymbol{H}_M \cdots \boldsymbol{H}_1 \boldsymbol{\Sigma}_{tx}^{1/2} &(13)\end{aligned}$$

The corresponding mutual information functions are given by

$$\begin{aligned}\mathcal{I}_{rx} &= \mathcal{I}(\gamma; \boldsymbol{\Sigma}_{rx}, \boldsymbol{H}_M \cdots \boldsymbol{H}_1) &(14)\\ \mathcal{I}_{tx} &= \mathcal{I}(\gamma; \boldsymbol{\Sigma}_{tx}, \boldsymbol{H}_1^{\mathsf{H}} \cdots \boldsymbol{H}_M^{\mathsf{H}}) &(15)\end{aligned}$$

where

$$\mathcal{I}(\gamma; \boldsymbol{\Sigma}_c, \boldsymbol{H}) \triangleq \log\det(\boldsymbol{I} + \gamma\boldsymbol{\Sigma}_c\boldsymbol{H}\boldsymbol{H}^{\mathsf{H}}). \quad (16)$$

The invariance in distribution of all the Ginibre matrices $\boldsymbol{H}_m$ with respect to the product by unitary matrices yields

$$\mathcal{I}(\gamma; \boldsymbol{\Sigma}_c, \boldsymbol{H}) = \mathcal{I}(\gamma; \boldsymbol{\Lambda}_c, \boldsymbol{H}) \quad (17)$$

where $\boldsymbol{\Lambda}_c$ is the diagonal matrix of the eigenvalues of $\boldsymbol{\Sigma}_c$. Now, if the size of the square covariance matrix $\boldsymbol{\Sigma}_c$ is $N_{\min}$, its eigenvalues are distinct, and $\boldsymbol{\Lambda}_c = \mathrm{diag}(\boldsymbol{\sigma})$, with $\boldsymbol{\sigma} = (\sigma_1, \ldots, \sigma_{N_{\min}})$, the corresponding MGF is given by

$$\Phi_{\mathcal{I}}(s; \boldsymbol{\Lambda}_c) = \mathbb{E}[\det(\boldsymbol{I} + \gamma\boldsymbol{\Lambda}_c\boldsymbol{U}_H\boldsymbol{\Lambda}_H\boldsymbol{U}_H^{\mathsf{H}})^{-s}], \quad (18)$$

where $\boldsymbol{U}_H$ is a unitary matrix independent of $\boldsymbol{\Lambda}_H = \mathrm{diag}(\boldsymbol{\lambda})$ and $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_{N_{\min}})$. Then, applying eq. (4) and [18, eqs. (47-48)], we obtain eq. (19). The derivation of (19) and the extension to the case when the dimension of $\boldsymbol{\Lambda}_c$ is greater than $N_{\min}$ are reported in a full paper in preparation.

$$\begin{aligned}\Phi_{\mathcal{I}}(s; \boldsymbol{\Lambda}_c) = & \frac{\prod_{k=1}^{N_{\min}-1}(N_{\min}-k)^k}{\gamma^{N_{\min}(N_{\min}+1)/2}\det(\boldsymbol{\Lambda}_c)\Delta(\boldsymbol{\sigma})\prod_{k=1}^{N_{\min}}\prod_{m=0}^{M}(k+\nu_m-1)!\prod_{k=1}^{N_{\min}-1}(s-k)^k} \\ & \det\left[\frac{1}{\Gamma(s-N_{\min}+1)} G_{1,M+1}^{M+1,1}\left(\begin{matrix} 0 \\ s-N_{\min}, \nu_M, \ldots, \nu_2, \nu_1 + j - 1\end{matrix}\middle|\frac{1}{\gamma\sigma_i}\right)\right]_{i,j=1}^{N_{\min}} \quad (19)\end{aligned}$$

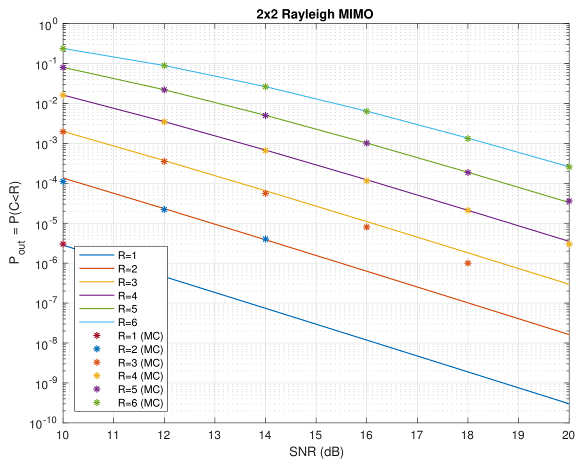

Fig. 1. Outage probability of a 2×2 Rayleigh MIMO channel versus the SNR in dB corresponding to different target rates $R = 1$ to 6 bit/s/Hz. Monte-Carlo simulation results are reported as markers for comparison purposes.

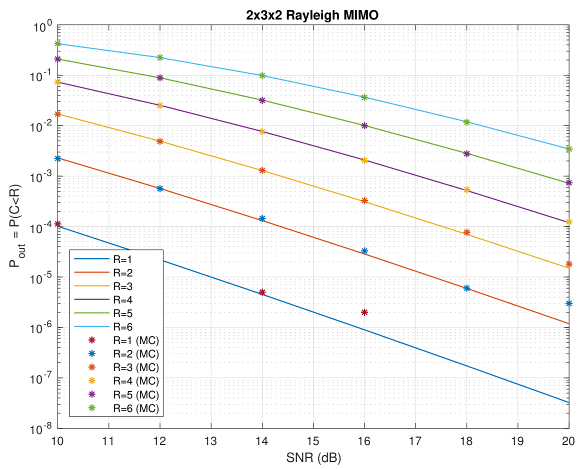

Fig. 2. Same as Fig. 1 but for a $2 \times 3 \times 2$ multi-cluster Rayleigh MIMO channel.

IV. Numerical results

In order to test the validity of the analytic results obtained in the previous sections we consider a MIMO channel with $n_T = n_R = 2$ transmit and receive antennas and a variable number of intermediate clusters (from 0 to 3) each of them containing $n_S = 3$ scatterers. The MIMO channels obtained in this way are denoted as $2\times\underbrace{3\times\cdots\times3}_{M-1}\times2$ Rayleigh MIMO channels for $M = 1, 2, 3, 4$. The results obtained by our analytic approach and by Monte-Carlo simulation are reported in Figs. 1 to 4.

We observe that the Monte-Carlo simulation results exhibit an excellent agreement for outage probabilities as low as 10^{-4}, which is a consequence of the fact that they are based on 10^6 samples. Moreover, we notice that increasing the number of intermediate clusters progressively increases the outage probability for given SNR and target rate R. However, the rate of increase reduces as the number of intermediate clusters increases.

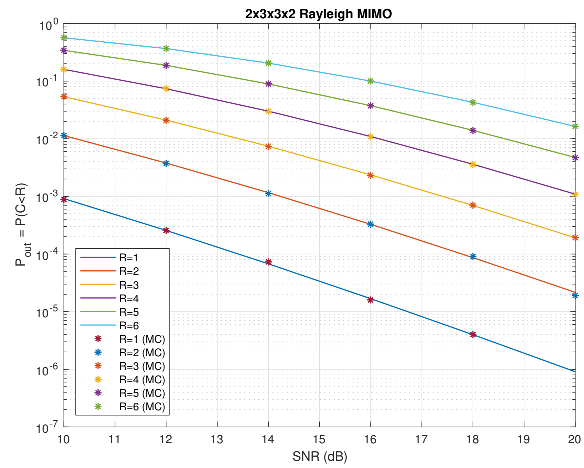

Fig. 3. Same as Fig. 1 but for a $2 \times 3 \times 3 \times 2$ multi-cluster Rayleigh MIMO channel.

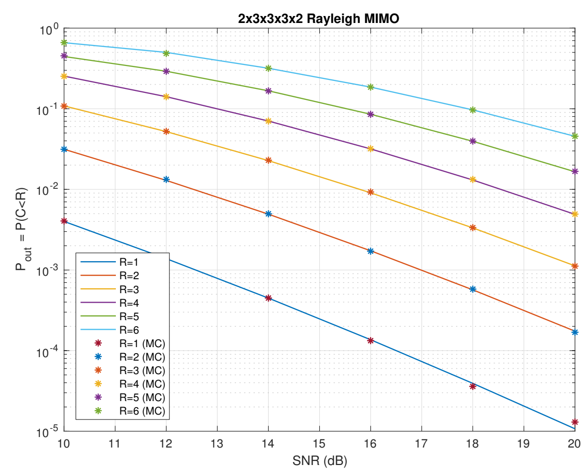

Fig. 4. Same as Fig. 1 but for a $2 \times 3 \times 3 \times 3 \times 2$ multi-cluster Rayleigh MIMO channel.

The effect of spatial correlation is evidenced in Figs. 5 to 7 assuming the spatial correlation matrix to be $\boldsymbol{\Sigma}_c = (0.8^{(i-1)(j-1)})_{i,j=1}^2$. Again, Monte-Carlo simulation results (based on 10^6 samples) are in excellent agreement with analytic results for outage probabilities as low as 10^{-4}. Spatial correlation introduces an overall degradation of the outage probability ranging from 1 to 2 dB (approximately) in the SNR scale.

V. Conclusions

We addressed the evaluation of the MGF of the information rate of multi-cluster scattering MIMO channel in the spatially uncorrelated case and in the one-sided spatially correlated case. The results allowed us to obtain an analytic expression for the outage probability based on determinants of Meijer functions. Monte-Carlo simulation was used to confirm the validity of the analytic results in some cases of interest.

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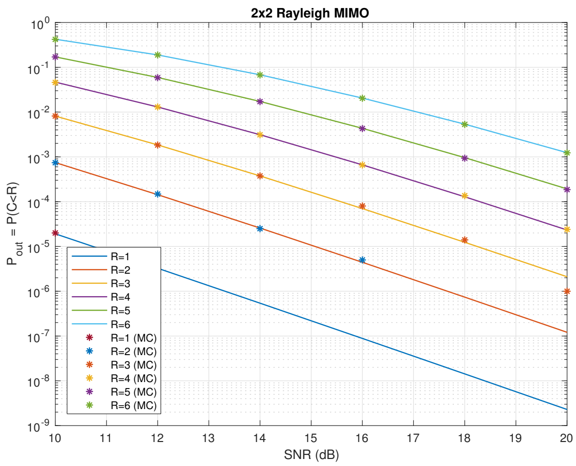

Fig. 5. Same as Fig. 1 with correlation matrix $\mathbf{\Sigma}_c = (0.8^{(i-1)(j-1)})_{i,j=1}^2$.

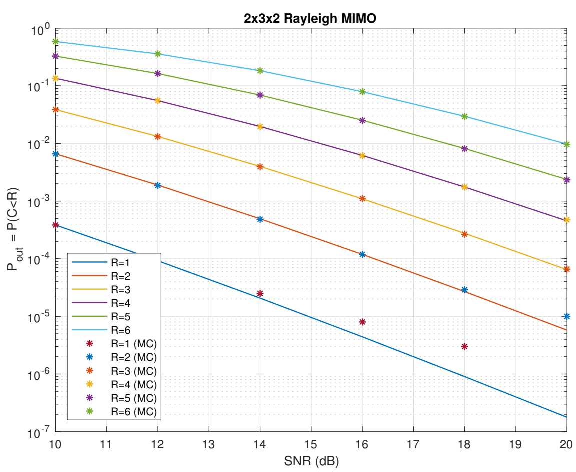

Fig. 6. Same as Fig. 2 with correlation matrix $\mathbf{\Sigma}_c = (0.8^{(i-1)(j-1)})_{i,j=1}^2$.

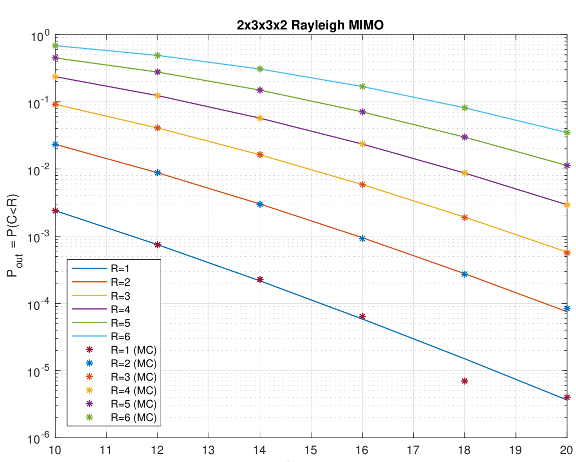

Fig. 7. Same as Fig. 3 with correlation matrix $\mathbf{\Sigma}_c = (0.8^{(i-1)(j-1)})_{i,j=1}^2$.

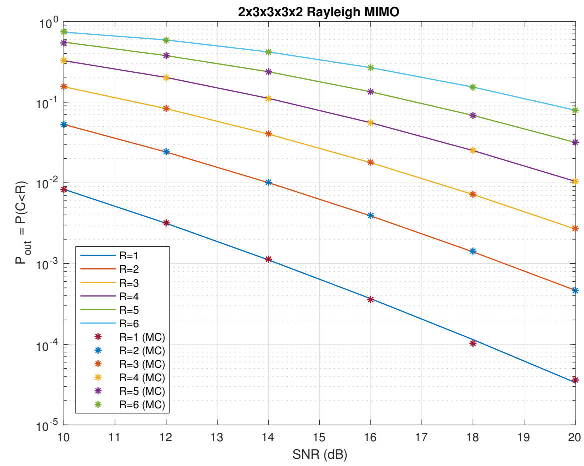

Fig. 8. Same as Fig. 4 with correlation matrix $\mathbf{\Sigma}_c = (0.8^{(i-1)(j-1)})_{i,j=1}^2$.

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